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Review
Mechanical properties of ice and snow

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The mechanical properties of ice and snow are reviewed. The tensile strength of ice varies from 0.7–3.1 MPa and the compressive strength varies from 5–25 MPa over the temperature range −10°C to −20°C. The ice compressive strength increases with decreasing temperature and increasing strain rate, but ice tensile strength is relatively insensitive to these variables. The tensile strength of ice decreases with increasing ice grain size. The strength of ice decreases with increasing volume, and the estimated Weibull modulus is 5. The fracture toughness of ice is in the range of 50–150 kPa $m^{1/2}$ and the fracture-initiating flaw size is similar to the grain size. Ice-soil composite mixtures are both stronger and tougher than ice alone. Snow is a open cellular form of ice. Both the strength and fracture toughness of snow are substantially lower than those of ice. Fracture-initiating flaw sizes in snow appear to correlate to the snow cell size. © *2003 Kluwer Academic Publishers*

1. Introduction

The mechanical properties of ice and snow are important to a number of diverse aspects. The area of glaciology requires such information in order to predict the movement and breakup of glaciers over time [1]. These properties are of key importance for the prediction and abatement of avalanche hazards [2]. The impingement of ice onto static man-made structures in cold regions requires information on mechanical properties [3]. Finally, ice and snow-like structures are thought to be the primary constituents of comet nuclei [4]. The purpose of the present review is to summarize and interpret the information that currently exists on the mechanical properties of ice and snow. The emphasis will be on freshwater ice and snow.

2. Ice

2.1. Ice crystal structure

Ice exists in a number of different crystal structures, as well as two amorphous states [5]. At low pressures, the stable phase is termed ice I. Ice I has two variants. Ice Ih is hexagonal and is obtained by the freezing of water at ambient pressure. Ice Ic is cubic and is formed by vapor deposition at low temperatures.

2.2. Ice elastic modulus

The elastic modulus and Poisson's ratio of polycrystalline ice has been measured by subjecting plates of ice to biaxial bending [6]. At a temperature of −10°C for measurements on ice plates that were 0.5 m in diameter, the Young's modulus of ice was reported in the range of 9.7–11.2 GPa and Poisson's ratio was 0.29–0.32.

2.3. Ice tensile and compressive strength

The strength of ice has been measured by a relatively small number of investigators [3, 5, 7–12]. There is a relatively wide range of scatter of ice tensile strength, from 0.7 MPa to 3.1 MPa. The average tensile strength of ice from published investigations is 1.43 MPa in the temperature range −10 to −20°C. Over this temperature range, the compressive strength of ice ranges between 5–25 MPa [7]. Ice strength depends on the variables of temperature, strain rate, tested volume, and ice grain size. These dependencies will now be discussed.

2.3.1. Effects of temperature

Generally, the strength of ice increases with decreasing temperature in both tension and compression, as shown in Fig. 1. This temperature effect on strength is more prominent in compression than in tension. Haynes reported [7] that the compressive strength of ice increased by approximately a factor of 4 from 0°C to −40°C. However, he indicated that the tensile strength of ice increased by only a factor of 1.3 over the same temperature range. Schulson has suggested that the temperature dependence of compressive strength of ice is related to ice dislocation and grain boundary sliding phenomena that lead to temperature-dependent damage accumulation [5]. The much more limited temperature dependence of tensile strength is related to the localization of stress-accommodating mechanisms at the tips of tensile flaws.

2.3.2. Effects of strain rate

Fig. 2 shows the effects of strain rate on the tensile and compressive strength of ice [5]. While the compressive strength is strain rate sensitive, the tensile strength is

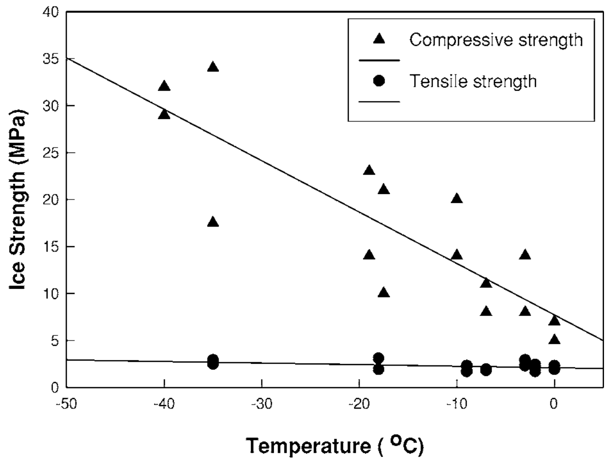

Figure 1 Tensile and compressive strength of ice as a function of temperature [7].

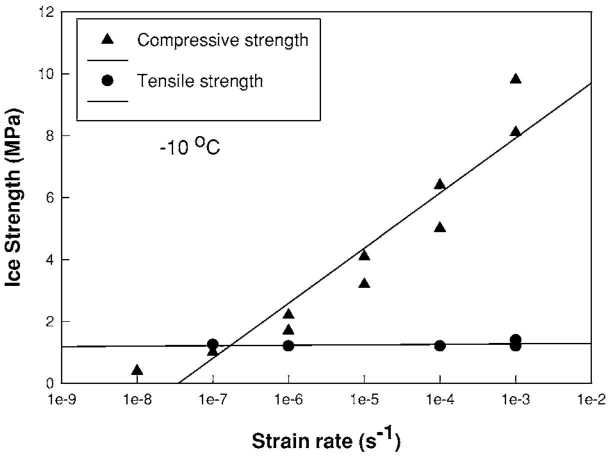

Figure 2 Tensile and compressive strength of ice as a function of strain rate [5].

strain rate insensitive, over the range of strain rates examined. Tensile stress-strain curves exhibit ductile behavior at low strain rates, but brittle behavior at intermediate and high strain rates. Compressive stress-strain curves show ductile behavior at low and intermediate strain rates, but brittle behavior at high strain rates. The strain rate effects are consistent with dislocation and grain boundary sliding deformation mechanisms that operate during the creep of ice [5, 13, 14].

2.3.3. Effects of grain size

The tensile strength of ice decreases with increasing ice grain diameter [8] as shown in Fig. 3. These data are well-described by a Hall-Petch type of relationship:

$$\sigma_f = \sigma_i + kd^n \tag{1}$$

where the exponent $n = -1/2$. This $d^{-1/2}$ dependence suggests that the tensile strength of ice is controlled of a stress concentration process. A possible process is the propagation of microcracks that are nucleated by dislocation pile-ups against grain boundaries [8].

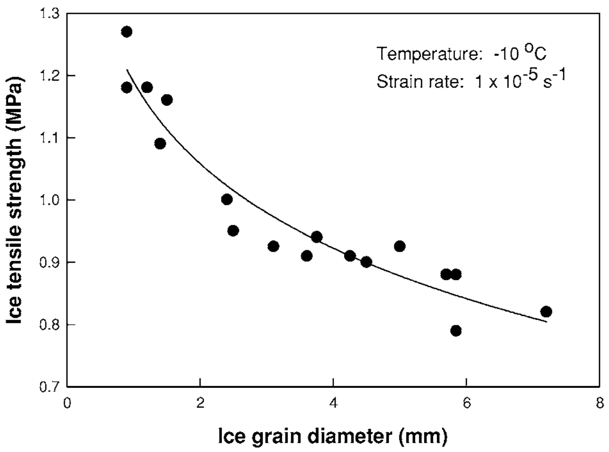

Figure 3 Tensile strength of ice as a function of grain size [8].

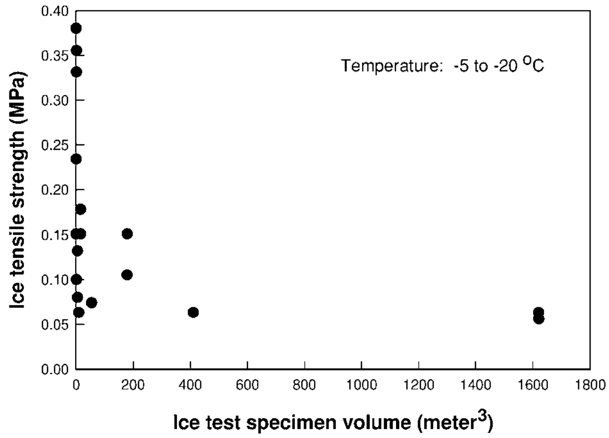

Figure 4 Tensile strength of ice as a function of volume [15, 16].

2.3.4. Effects of volume

The tensile strength of ice decreases with increasing test specimen volume [15, 16] as shown in Fig. 4. Volume effects on the strength of brittle materials are usually described by a Weibull statistical distribution approach [17]. The Weibull theory is often referred to as a "weakest-link" theory of fracture. In the Weibull theory, the probability of fracture is given by:

$$\mathrm{P} = 1 - \exp\left(-v(\sigma/\sigma_o)^m\right) \tag{2}$$

where P = probability of fracture, σ = applied tensile stress (which is assumed to be uniform over the stressed volume of the material), σ_o = a constant, v = stressed volume, and m = Weibull modulus. This expression leads to the following expression for the volume dependence of the strength of brittle materials [18]:

$$\sigma_2/\sigma_1 = (v_1/v_2)^{1/m} \tag{3}$$

From the strength-volume data in Fig. 4, it was possible to obtain a Weibull modulus value for ice. The Weibull modulus of ice is estimated to have a value of approximately 5. To our knowledge, this is the first time a Weibull modulus for ice has ever been put forward. By way of comparison, the Weibull modulus of brittle ceramic materials is typically in the range of 5–20. The higher the value of the Weibull modulus, the lower is the statistical scatter in the measured fracture stress.

2.4. Fracture toughness

The fracture toughness of ice has seen only limited investigation [1, 19–25]. Generally, the fracture toughness of ice is in the range of 50–150 kPa m$^{1/2}$. By way of comparison, the fracture toughness of glass is typically 700–1000 kPa m$^{1/2}$ [26]. Thus, ice has roughly one-tenth the fracture toughness of glass.

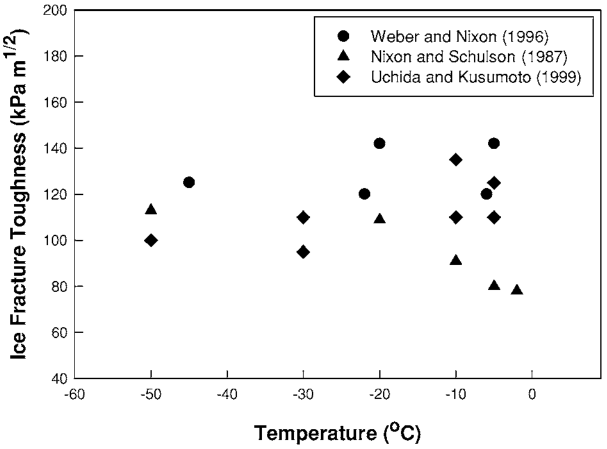

Figure 5 Fracture toughness of ice as a function of temperature [20, 24, 25].

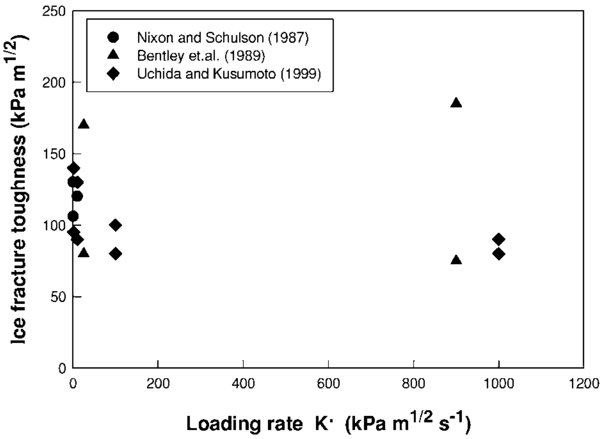

Figure 6 Fracture toughness of ice as a function of loading rate [20, 22, 25].

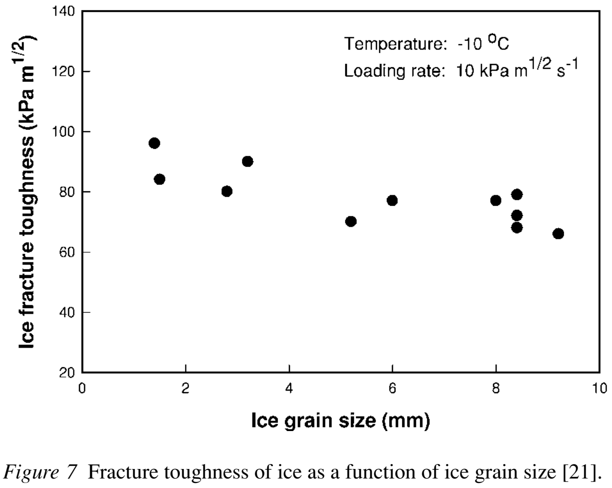

Figure 7 Fracture toughness of ice as a function of ice grain size [21].

The fracture toughness of ice is shown as a function of temperature [20, 24, 25] in Fig. 5. Ice fracture toughness appears to be a relatively weak function of temperature. The fracture toughness of ice as a function of loading rate [20, 22, 25] is shown in Fig. 6. Ice fracture toughness is relatively insensitive to loading rate. The fracture toughness of ice as a function of ice grain size [21] is shown in Fig. 7. The fracture toughness exhibits a decrease with increasing grain size. Nixon and Schulson [20] have suggested that a microcrack toughening mechanism may be operative in ice.

2.5. Fracture-initiating defect size

One may use average values of the fracture strength and fracture toughness of ice to obtain an approximate value of the fracture-initiating defect size. For fracture from a penny-shaped internal flaw in a large body subjected to uniaxial tension, the relationship between the fracture strength, fracture toughness, and fracture flaw size is [27]:

$$K_{c} = (1.128)(\sigma_{f})(a)^{1/2} \qquad (4)$$

Here "a" is the flaw radius (flaw diameter $= 2a$).

Using an average fracture strength of ice of 1.43 MPa and an average ice fracture toughness of 0.1 MPa m$^{1/2}$, the calculated flaw diameter of the average fracture initiating defects is 7.7 mm. This value is in the range of typical grain diameters for ice microstructures [21].

2.6. Thermal shock

The thermal shock of ice has been investigated [28]. Ice spheres 2–3 cm in diameter were cooled to various sub-zero temperatures, then rapidly heated in water at 0°C. Both clear ice and ice with internal bubbles were examined. The thermal shock fracture probability was observed to be 50% at a temperature difference of 15°C and 100% at a temperature difference of 20°C and higher. The thermal shock cracks formed below the sphere surface. Cracks were spherical in shape (although not completely closed spheres) and were concentric with the outer spherical surface. All of the ice spheres examined remained intact after thermal shock cracking.

2.7. Ice-soil mixtures

A small number of studies have been conducted on ice-soil mixtures [29–33]. These materials effectively constitute composite materials of ice and soil. The ice-soil mixtures in these investigations contained approximately 50 vol% soil and 50 vol% ice.

The strength and fracture toughness of these ice-soil mixtures as a function of temperature are shown in Figs 8 and 9, respectively. Comparison of these data to the strength and fracture toughness of ice shows that the ice-soil mixtures are both substantially stronger and

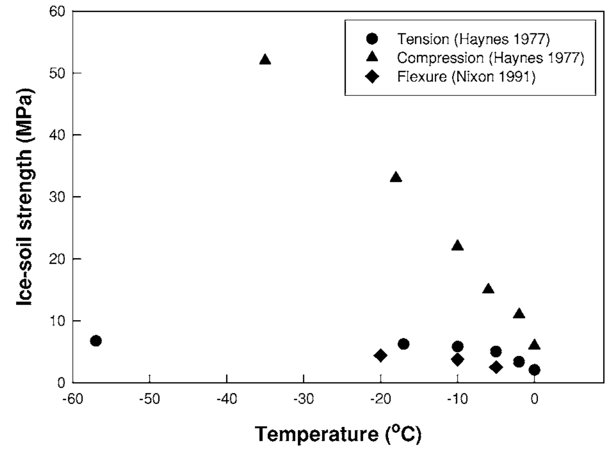

Figure 8 Strength of ice-soil mixtures as a function of temperature [29, 31].

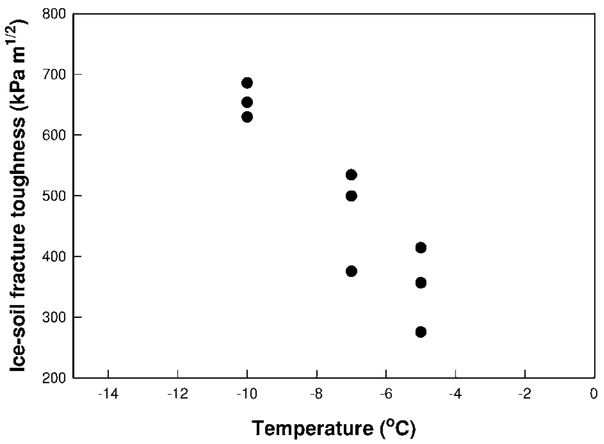

Figure 9 Fracture toughness of ice-soil mixture as a function of temperature [32].

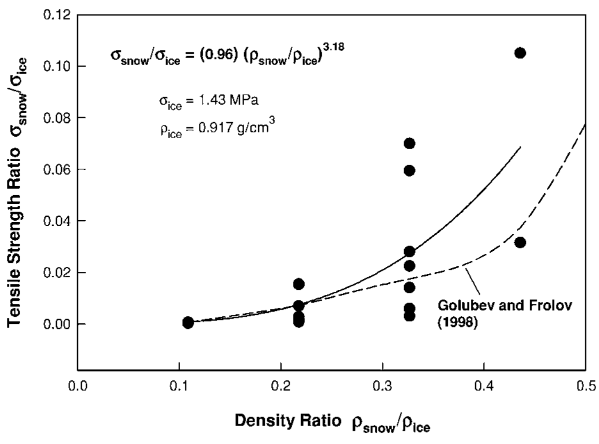

Figure 11 Snow/ice tensile strength ratio versus snow/ice density ratio.

more fracture-resistant than ice alone. It has been reported [30] that the strength of ice-soil mixtures increases with increasing loading rate. A value of 20 MPa is indicated at a high strain rate of 10^4 s^{-1}.

3. Snow

3.1. Structure of snow

Snow may best be regarded as a cellular form of ice, in which the individual ice crystals of snow are bonded together. The mechanical properties of cellular solids have been described in some detail [34]. Cellular solids can be of either a closed cell form (e.g. soap foam) or an open cell form (e.g. sponge). Snow is of the open cell type where individual ice particles bonded in linear chains form an open cell polyhedral-type structure.

3.2. Tensile strength of snow

The mechanical properties of snow have been investigated to a limited extent [2, 35–47]. The tensile strength of snow as a function of density [35, 36, 40] is shown in Fig. 10. As may be seen, the tensile strength of snow is much lower than that of ice, decreasing substantially with decreasing snow density. The Weibull modulus of snow has been reported to be in the range of 0.9–1.6 and independent of snow density [35].

Using the snow strength data in Fig. 10, the snow/ice tensile strength ratio is plotted as a function of snow/ice density ratio in Fig. 11. It is observed that the tensile strength ratio is approximately proportional to the cube of the density ratio. Based on a cellular model for snow, a model has been developed relating the microstructural features of snow to its macroscopic tensile strength [43]. This model suggests that snow strength is dependent primarily on the bonding contact of individual ice particles, their shape, and the number of particle contacts with neighboring particles. The predictions of this model are shown in Fig. 11. As may be seen, the model provides an underestimate of the strength of snow.

3.3. Fracture toughness of snow

The fracture toughness of snow as a function of density [45] is shown in Fig. 12. The fracture toughness of snow is exceedingly small, being approximately 2–3 orders of magnitude lower than the fracture toughness of ice.

Using the snow fracture toughness data in Fig. 12, the snow/ice fracture toughness ratio is plotted as a function of snow/ice density ratio in Fig. 13. The fracture toughness ratio is approximately proportional to the square of the density ratio. A model of the fracture toughness of open cell foams predicts that the fracture toughness of these materials will be proportional to the density ratio to the 3/2 power [48].

3.4. Fracture-initiating defect sizes in snow

From the measured strength and fracture toughness values of snow, the size of the fracture initiating flaw size in snow can be calculated via Equation 4. The flaw

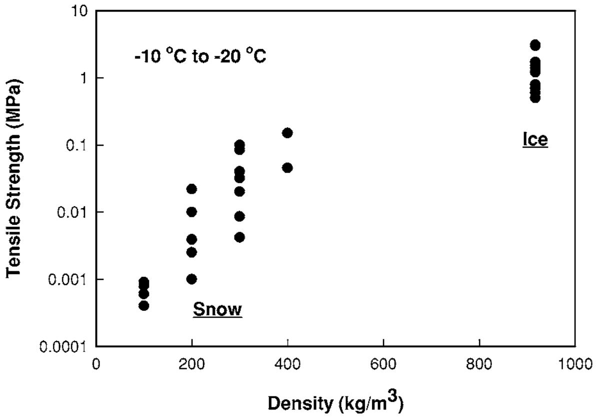

Figure 10 Tensile strength of snow as a function of snow density [35, 36, 40].

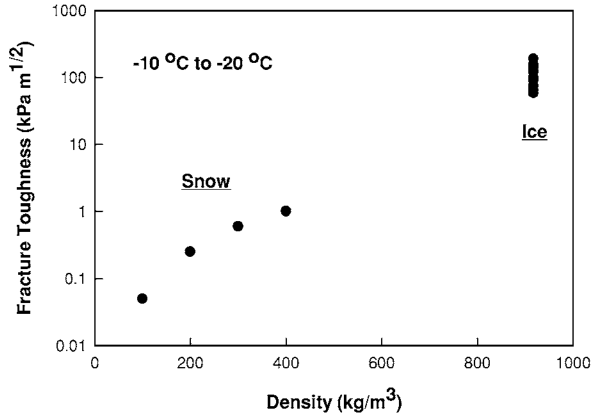

Figure 12 Fracture toughness of snow as a function of snow density [45].

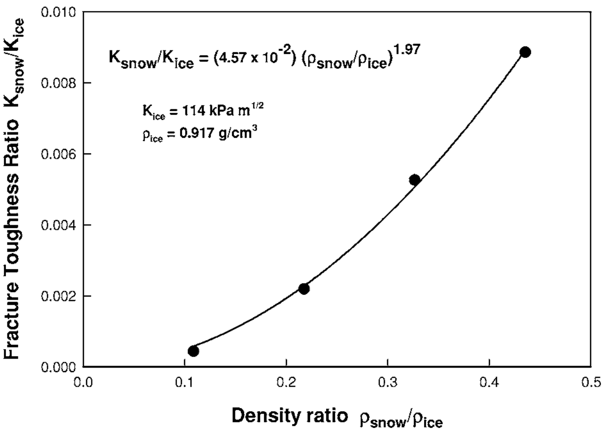

Figure 13 Snow/ice fracture toughness ratio as a function of snow/ice density ratio.

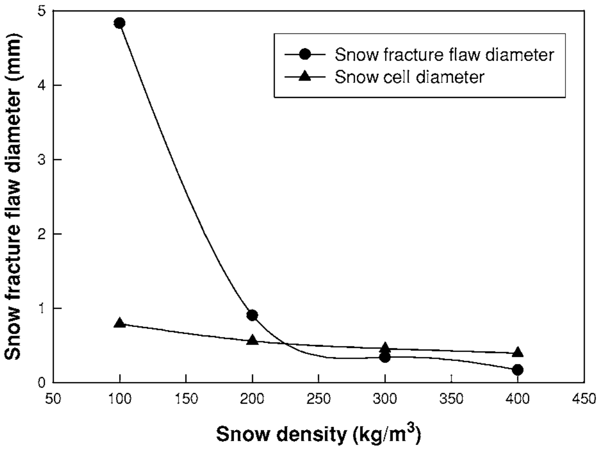

Figure 14 Fracture flaw diameter of snow as a function of snow density. A comparison is made to the calculated snow cell diameter.

diameter of snow as a function of snow density is shown in Fig. 14.

It is interesting to note that the snow fracture flaw sizes are considerably lower than the fracture flaw size of 7.7 mm obtained for ice, except for the lowest snow density. This would suggest that the cellular nature of snow has an effect on the flaw size for brittle fracture in that the fracture flaw size becomes associated with the cell size of the individual cells. The cell diameter of snow was calculated as a function of snow density. It was assumed that the snow cells were open cell tetrakaidecahedra [34] and that the cell strut thickness was 0.1 mm [2]. The comparison of snow fracture flaw diameter to snow cell diameter is shown in Fig. 14. A good correspondence exists except at the lowest snow density. At snow densities closer to that of ice, one would expect a transition from cell size fracture to grain size fracture, with an increase in the fracture flaw size. However, no data exist on both snow strength and fracture toughness for the higher snow density region.

4. Conclusions

The mechanical behavior of ice exhibits a similarity to the mechanical behavior of brittle ceramics. However ice strength, elastic properties, and fracture toughness are all significantly lower than ceramic materials. These differences in mechanical property levels are related to differences in the atomic bonding operative in ice as compared to atomic bonding in ceramics. Fundamental studies along these lines may be insightful. Interestingly, the tensile strength of ice is relatively insensitive to temperature and strain rate, while the compressive strength is dependent on these variables. For ceramic materials, such a difference in the sensitivity of tensile and compressive strength to these variables is typically not observed. In ceramics, either both tensile and compressive ceramic strengths are temperature and strain rate sensitive, or both are not. This suggests a difference in tensile-compressive fracture mechanisms in ice as compared to these mechanisms in ceramic materials. Additional research is required to more comprehensively describe the deformation mechanisms operative in ice in both tension and compression. It is important to note that the strength and fracture toughness of ice-soil mixtures are greater than those of ice alone. This strongly suggests the operation of composite-type behavior, with ice as the matrix and soil as the reinforcement. The application of ceramic composite methodologies to ice may prove to be a fruitful area for improving the mechanical properties of ice-based structures.

Snow, the cellular form of ice, is indeed a fragile material in terms of its mechanical properties. Snow exhibits some of the lowest levels of strength and fracture toughness known for commonly encountered material forms. Very little information on the mechanical properties of snow exists, and almost no work has been done to relate this mechanical behavior to key snow microstructural variables. Microstructure-property relationships being developed for cellular materials may provide new insights into the mechanical behavior of snow and snow structures.

References

1. R. M. ANDREWS, *Journal of Glaciology* **31** (1985) 171.
2. M. MELLOR, *ibid.* **19** (1977) 15.
3. R. L. HOOKE *et al.*, *Cold Regions Science and Technology* **3** (1980) 263.
4. J. M. GREENBERG, *Astronomy and Astrophysics* **330** (1998) 375.
5. E. M. SCHULSON, *Journal of the Minerals, Metals, Materials Society* **51** (1999) 21.
6. L. W. GOLD, *Canadian Journal of Civil Engineering* **15** (1988) 1080.
7. F. D. HAYNES, "Effect of Temperature on the Strength of Snow-Ice," Department of the Army, Cold Regions Research and Engineering Laboratory, Corps of Engineers, CRREL Report 78-27, Hanover, New Hampshire, December 1978.
8. J. H. CURRIER and E. M. SCHULSON, *Acta metal.* **30** (1982) 1511.
9. R. W. LEE and E. M. SCHULSON, *Journal of Offshore Mechanics and Arctic Engineering* **110** (1988) 187.
10. X. XIAN, M. L. CHU, R. J. SCAVUZZO and T. S. SRIVATSAN, *Journal Mater. Sci. Lett.* **8** (1989) 1205.
11. J. DRUEZ and P. McCOMBER, *Transactions of the Canadian Society for Mechanical Engineering* **13** (1989) 59.
12. E. M. SCHULSON, S. G. HOXIE and W. A. NIXON, *Philosophical Magazine A* **59** (1989) 303.
13. P. DUVAL, M. F. ASHBY and I. ANDERMAN, *J. Phys. Chem* **87** (1983) 4066.
14. J. WEERTMAN, *Annu. Rev. Earth Planet Sci.* **11** (1983) 215.
15. J. P. DEMPSEY, S. J. DEFRANCO, R. M. ADAMSON and

S. V. MULMULE, *International Journal of Fracture* **95** (1999) 325.

16. J. P. DEMPSEY, R. M. ADAMSON and S. V. MULMULE, *ibid.* **95** (1999) 347.
17. W. WEIBULL, *Ingenioersvetenskapsakad., Handl.* **151** (1939) 1.
18. J. J. PETROVIC, *Metallurgical Transactions A* **18A** (1987) 1829.
19. R. M. ANDREWS, *Journal of Glaciology* **31** (1985) 171.
20. W. A. NIXON and E. M. SCHULSON, *Journal de Physique* **48** (1987) C1-313.
21. *Idem.*, *Journal of Offshore Mechanics and Arctic Engineering* **110** (1988) 192.
22. D. L. BENTLEY, J. P. DEMPSEY, D. S. SODHI and Y. WEI, *Cold Regions Science and Technology* **17** (1989) 7.
23. M. P. FISCHER, R. B. ALLEY and T. ENGELDER, *Journal of Glaciology* **41** (1995) 138.
24. L. J. WEBER and W. A. NIXON, *Journal of Offshore Mechanics and Arctic Engineering* **118** (1996) 135.
25. T. UCHIDA and S. KUSUMOTO, *JSME International Journal Series A – Solid Mechanics and Materials* **42** (1999) 601.
26. G. R. ANSTIS, P. CHANTIKUL, B. R. LAWN and D. B. MARSHALL, *Journal of the American Ceramic Society* **64** (1981) 533.
27. B. LAWN, "Fracture of Brittle Solids," 2nd ed. (Cambridge University Press, Cambridge, UK, 1993.).
28. W. D. KING and N. H. FLETCHER, *Journal of the Atmospheric Sciences* **33** (1976) 97.
29. F. D. HAYNES and J. A. KARALIUS, "Effect of Temperature on the Strength of Frozen Silt," Cold Regions Research and Engineering Laboratory Report CRREL 77-3, U.S. Army Corps of Engineers, Hanover, New Hampshire, February 1977.
30. M. A. LANGE and T. J. AHRENS, *Journal of Geophysical Research* **88** (1983) 1197.
31. W. A. NIXON and L. J. WEBER, *Journal of Cold Regions Engineering* **5** (1991) 14.
32. H. LI and H. YANG, *ibid.* **14** (2000) 43.
33. H. LI, H. YANG and Z. LIU, *Can. Geotech. J.* **37** (2000) 253.
34. L. J. GIBSON and M. F. ASHBY, "Cellular Solids," 2nd ed. (Cambridge University Press, 1997.).
35. R. A. SOMMERFELD, *Journal of Geophysical Research* **79** (1974) 3353.
36. D. M. McCLUNG, *Journal of Glaciology* **22** (1979) 1.
37. Z. WATANABE, *ibid.* **26** (1980) 255.
38. H. NARITA, *ibid.* **26** (1980) 275.
39. B. SALM, *Reviews of Geophysics and Space Physics* **20** (1982) 1.
40. H. NARITA, *Contributions From the Institute of Low Temperature Sciences, Series A* **32** (1983) 1.
41. J. B. JOHNSON, D. J. SOLIE, J. A. BROWN, and E. S. GAFFNEY, *J. Appl. Phys.* **73** (1993) 4852.
42. K. C. AGRAWAL and R. K. MITTAL, *Defence Science Journal* **45** (1995) 93.
43. V. N. GOLUBEV and A. D. FROLOV, *Annals of Glaciology* **26** (1998) 45.
44. R. F. TRUNIN, G. V. SIMAKOV, M. V. ZHERNOKLETOY and V. V. DOROKHIN, *High Temperature* **37** (1999) 702.
45. H. O. K. KIRCHNER, G. MICHOT and T. SUZUKI, *Philosophical Magazine A* **80** (2000) 1265.
46. V. N. GOLUBEV and A. D. FROLOV, *Annals of Glaciology* **31** (2000) 434.
47. H. O. K. KIRCHNER, G. MICHOT and J. SCHWEIZER, *Scripta Materialia* **46** (2002) 425.
48. S. K. MAITI, M. F. ASHBY and L. J. GIBSON, *ibid.* **18** (1984) 213.

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